

Propellant Storage Considerations for Electric Propulsion

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Abstract

Propellant storage considerations which are particular to electric propulsion space missions are examined. Relations are derived between tank heat loading and propulsion power requirements for both ion engine and arcjet missions using cryogenic storage. Cryogenic storage of argon, krypton, and xenon is shown to be possible for appropriate missions. Cryogenic storage of hydrogen and helium for arcjets is shown to be both necessary and possible, with tankage fractions less than 20%. Supercritical storage of the noble gases for ion engines is possible, with reasonable tankage fractions for at least xenon and krypton. It is shown that there is an optimum storage pressure specific to each gas which will minimize the tank mass. Deviations from this optimum pressure cause significant mass penalties. In particular, using smaller, higher pressure tanks because of spacecraft volume constraints is shown to be of very limited utility.

Introduction

With the recent resurgence of interest in electric propulsion (EP), there is a new need for a general review of the propellant storage problems specific to this field. The subject was examined fairly extensively in the early sixties¹⁻⁵ but, for two reasons, a new survey of the field is appropriate. First, most of the work done in the sixties appeared only as corporate or NASA technical reports and is not widely available. Second, much of the old work dealt with specific tank designs, and has been rendered obsolete by advances in technology in areas such as insulation and tank materials, and by the consensus reached in the past decade that the noble gases are more suitable than mercury or cesium as propellants for ion engines. My purpose is not to present specific tank designs, or to examine all the related technologies. I simply wish to outline the basic principles which act as fundamental limits to the

mass of EP propellant storage systems, and to make this information available as a starting point for engineers who may be considering the relative merits of various propulsion systems.

Storage considerations specific to electric propulsion application fall mostly in two areas, either long term cryogenic storage with a slow but steady consumption, or light-weight high-pressure tanks for supercritical gases. Cryogenic storage for electric propulsion differs from other space applications of cryogenics in two ways. The storage period is long compared to chemical cryogenic propulsion missions such as the Centaur upper stage, but in comparison with space-based cryogenic cooling applications, such as COBE, the use rate in EP applications is high, and the feed can be a room temperature gas, allowing the propellant flow to cool the tank. Supercritical gas storage for EP differs from other space-based supercritical storage applications in the nature of the gas, and in that the tank and gas constitute a very large fraction of the spacecraft mass, enhancing the need to minimize the tankage fraction. In this paper, I will review the structural considerations associated with pressure containment and acceleration loading. I will then examine the technology of cryogenic storage and insulation in space.

Propellants

A brief review of the potential propellants is in order first. As mentioned, the currently favored propellants for ion engines are the noble gases: argon, krypton, and xenon. From the standpoint of storability and engine efficiency, the heavier gases are preferred over the lighter, but krypton and particularly xenon are scarce and expensive enough that their cost can be a serious concern,⁶ leading to possible use of argon for missions requiring very large amounts of propellant, for example, a manned Mars mission. Each of these propellants can be stored either as a high pressure su-

Table 1. Properties of selected propellants.

	Hydrogen	Helium	Argon	Krypton	Xenon
Normal Boiling Point (K)	20.384	4.125	87.29	119.8	165.0
Density at NBP (kg/m ³)	67	124.8	1392	2412	3080
Critical Temperature (K)	33.24	5.20	150.7	209.4	289.74
Critical Pressure (kPa)	1297	229	4864	5490	5838
Heat of Vaporization at NBP (J/g)	445	21.0	163	107.5	96.1

percritical gas or as a cryogenic liquid, and the method selected will depend on the mission. Because of the various missions, both the cryogenic and supercritical properties of these propellants must be examined.

For arcjets, propellants which are given serious consideration are ammonia, hydrazine, and hydrogen. Helium also shows some promise as an arcjet propellant, but is usually dismissed as being too difficult to store. Ammonia and hydrazine can be stored as liquids at ambient temperatures on spacecraft, while hydrogen, because of its low density and low boiling point, must be stored as a cryogenic liquid. The same, of course, applies to helium. Table One lists various properties of all of these propellants. Because ammonia and hydrazine are considered to be "space storable" propellants, and because they have been used extensively for other space applications, they present no storage problems specific to electric propulsion, and will not be dealt with in this paper.

In the case of propellants stored at ambient temperature, the tank needs to satisfy only two requirements; it must be able to contain the internal pressure, and it must be able to support its own weight and the weight of the propellants against launch loads. When the propellant must be carried as a cryogenic liquid, the tank must also satisfy requirements for thermal protection. Since structural requirements are common to all tanks, I will examine that area first.

Two additional points are useful to keep in mind for background information. First, tank configurations commonly used in space include spherical and cylindrical, so I will limit my analysis to these two shapes. Flat surfaces, of course, are not useful for pressure containment. Second, for solar powered EP missions, spacecraft orientation constraints will be driven by the need to keep the solar panels pointed at the sun and the thruster pointed along the thrust vector, so it will be difficult to attempt any propellant tank cooling by try-

ing to keep the tank on the shady side of the spacecraft. Equilibrium black body temperature in low Earth orbit (LEO) is about 315 K,¹ and one must be conservative and assume that ambient spacecraft temperatures will be near 300 K. Of course, as the spacecraft moves away from LEO, the equilibrium temperature will go down, but in electrical orbital transfer missions the tanks will be full only in LEO, where the temperatures are highest. One must design the tanks for the worst case, so in the analyses which follow I will assume an ambient temperature of 300 K.

Supercritical Storage

Consider first the spherical tank. For a thin walled spherical shell containing a pressurized fluid, the tensile stress in the shell is $S = Pr/2t$, where t is the wall thickness, r is the radius of the inner surface, and P is the internal pressure. If we assume a safety factor β , then the maximum stress in the wall is limited to $S = \sigma_y/\beta$, where σ_y is the yield strength of the tank material. We thus obtain an expression for the minimum wall thickness required to contain the pressure

$$t = \frac{Pr\beta}{2\sigma_y} \quad (1)$$

The radius of the tank is determined by the mass and density of the propellant m_p required for the mission:

$$r = \left[\frac{3m_p}{4\pi\rho_p} \right]^{1/3} \quad (2)$$

where ρ_p is density of the propellant at the pressure P . The mass of the tank is then

$$m_t = 4\pi r^2 t \rho_t = \frac{3P\beta m_p \rho_t}{2\sigma_y \rho_p} \quad (3)$$

where ρ_t is the density of the tank material.

For a cylindrical tank with spherical endcaps, the circumferential tensile stress in the cylindrical wall segment is $S = Pr/t$, while the longitudinal tensile stress,

and the tensile stress in the endcaps, are both the same as the spherical tank: $S = Pr/2t$. Following the same reasoning as in the spherical case, we find that the wall thickness in the cylindrical portion of the tank must be

$$t_c = \frac{Pr\beta}{\sigma_y} \quad (4)$$

while in the endcaps it is the same as in the spherical case

$$t_e = \frac{Pr\beta}{2\sigma_y} \quad (5)$$

The volume of a cylindrical tank with spherical endcaps is

$$V = \frac{4}{3}\pi r^3 + \pi r^2 l = \pi r^2 \left(\frac{4r}{3} + l \right) \quad (6)$$

where l is the length of the cylindrical portion of the tank. The mass of the tank is then

$$m_t = \rho_t (4\pi r^2 t_e + 2\pi r l t_c) = \frac{2\pi P\beta r^2 \rho_t}{\sigma_y} (r + l) \quad (7)$$

For convenience, I will define a tank eccentricity factor $\gamma = (r + l)/r$. For a fixed volume (and pressure and density), the mass of the tank will vary as a function of the eccentricity:

$$\frac{m_t}{V} = \frac{2P\beta\rho_t(r+l)}{\sigma_y(\frac{4}{3}r+l)} = \frac{2P\beta\rho_t\gamma}{\sigma_y(\frac{1}{3}+\gamma)} \quad (8)$$

Since $V = m_p/\rho_p$, we have for a cylindrical tank

$$m_t = \frac{2P\beta m_p \rho_t \gamma}{\sigma_y \rho_p (\frac{1}{3} + \gamma)} \quad (9)$$

In the limit as $l \rightarrow 0$ (a spherical tank), $\gamma \rightarrow 1$ and we get

$$\frac{m_t}{V} = \frac{3P\beta\rho_t}{2\sigma_y} \quad (10)$$

as expected, and in the limit as $\gamma \rightarrow \infty$ (an infinitely long cylinder) we get

$$\frac{m_t}{V} = \frac{2P\beta\rho_t}{\sigma_y} \quad (11)$$

This shows that the structural mass of a cylindrical tank will be between 1 and 1.33 times the structural mass of a spherical tank of equal volume. A plot of the mass penalty is shown as a function of eccentricity in Figure 1. For an eccentricity of 2 the mass penalty is 14%, and for $\gamma = 4$ the penalty is up to 23%.

Now let us examine how the properties of the propellant influence decisions about storage pressure. The

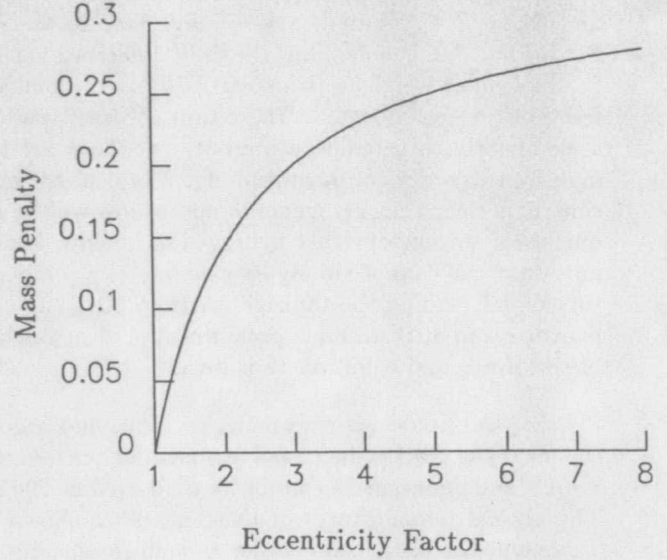


Figure 1. Tank mass penalty for cylindrical tanks, expressed as a fraction of an equivalent volume spherical tank.

relation between pressure and density for an ideal gas is

$$P = \rho RT/M \quad (12)$$

where R is the universal gas constant, T is the temperature, and M is the molecular weight of the gas. Substituting this expression for P in equations 3 and 9, we see that the tank mass is independent of the storage pressure or density. This allows one to make the storage tank smaller, within limits, to conserve space, without paying a mass penalty. The limits are encountered when the tank wall is no longer "thin," or when the gas is no longer "ideal." Of the propellants under consideration, only hydrogen and helium can be treated as ideal gases at ambient spacecraft temperatures.

For a thin walled spherical tank containing an ideal gas, the tank mass is

$$m_t = \frac{3P\beta m_p \rho_t}{2\sigma_y \rho_p} = \frac{3RT\beta m_p \rho_t}{2\sigma_y M} \quad (13)$$

The tankage fraction is defined as the ratio of the tank mass to the propellant mass, in this case

$$\frac{m_t}{m_p} = \frac{3RT\beta\rho_t}{2\sigma_y M} \quad (14)$$

As a representative case, consider the storage of hydrogen gas at 300 K in a spherical titanium tank. The maximum published yield strength for titanium is about $1.4 \times 10^9 \text{ N/m}^2$,⁷ and the density is 4.85 g/ml or 4850 kg/m³. The gas constant is 8314 J/(kg-mol K)

and $M = 2$ kg/kg-mol. Assuming $\beta = 2$, we get $m_t/m_p = 13.2$. For helium, the only difference is that the molecular weight is 4 instead of 2, so the tankage fraction is reduced to 6.6. These tankage fractions will scale linearly with temperature but, since they are too high by two orders of magnitude for a typical EP mission, it is clear that cryogenic temperatures will be required for storage of either hydrogen or helium. Please note that one cannot simply extrapolate to a temperature of 3 K to find the tankage fraction. At that temperature and at reasonable pressures, the ideal gas law breaks down and equation 12 is invalid.

For the noble gases (xenon, krypton, and argon) the ideal gas law begins to fail at much higher temperatures, and equation 12 cannot be used even at 300 K. The critical temperatures of these gases, as shown in Table One are of the same order as ambient spacecraft temperatures. To obtain tankage fractions for these gases, one must find a substitute for equation 12. The relation between pressure and density is not a simple one in the vicinity of the critical point, and has been modelled by several different equations. One which gives fairly good agreement with the experimental data is the Redlich-Kwong equation:

$$RT = (V_s - b) \left[P + \frac{a}{V_s(V_s + b)\sqrt{T}} \right] \quad (15)$$

In this expression, a and b are parameters which depend on the critical temperature and pressure of the gas of interest, and V_s is the specific volume, which is related to the density by $V_s = M/\rho_p$, with M being the molecular weight. Solving for P gives

$$P = \frac{RT\rho_p}{(M - b\rho_p)} - \frac{a\rho_p^2}{M(M + b\rho_p)\sqrt{T}} \quad (16)$$

Substituting this form of the pressure into equation 3, we obtain an expression for the tankage fraction as a function of the density of the propellant:

$$\frac{m_t}{m_p} = \frac{3\rho_t\beta}{2\sigma_y} \left[\frac{RT}{(M - b\rho_p)} - \frac{a\rho_p}{M(M + b\rho_p)\sqrt{T}} \right] \quad (17)$$

This expression shows that for a constant propellant density and temperature, the mass of the tank is linearly proportional to the mass of the propellant. Contrary to the ideal gas case, the tankage fraction here depends in a complicated way on the propellant density. A propellant density which will minimize the tankage fraction can be found by solving

$$\frac{\partial(m_t/m_p)}{\partial\rho_p} = 0 \quad (18)$$

After going through the algebra, we obtain

$$\rho_p = \frac{(4RT^{3/2}bM^2a)^{1/2} - RT^{3/2}bM - aM}{RT^{3/2}b^2 - ab} \quad (19)$$

where the Redlich-Kwong parameters are given by

$$a = \frac{R^2T_c^{5/2}}{9(2^{1/3} - 1)P_c} \quad b = \frac{(2^{1/3} - 1)RT_c}{3P_c} \quad (20)$$

T_c and P_c are the critical temperature and pressure of the gas. This expression for the optimum propellant density in a spherical tank depends only on the temperature and intrinsic properties of the gas being stored. It is independent of tank material, of tank size (within the "thin wall" limit), and of total propellant mass. For xenon at 300 K, the optimum density is about 1.35 g/ml at a pressure of 8.3 MPa; for krypton at 300 K, the optimum occurs at 0.8 g/ml and 17.9 MPa; for argon at 300 K, the optimum occurs at .25 g/ml and 14.8 MPa.

Recognizing that the quantity in square brackets in equation 17 is just P/ρ_p allows the tankage fraction to be written in a useful form:

$$\frac{m_t}{m_p} = \frac{3}{2} \frac{P\beta}{\sigma_y \rho_p} \quad (21)$$

where P is the storage pressure at density ρ_p . For the titanium tank mentioned above, and neglecting the safety factor (i.e. $\beta = 1$) the tankage fraction for xenon is 3.2%. With a safety factor of 2, the tankage fraction becomes 6.4%. For krypton, the corresponding numbers are 12% and 24%, and for argon, 31% and 62%.

In the case of the cylindrical tank, we can substitute equation 16 into equation 9 and obtain

$$\frac{m_t}{m_p} = \frac{2\rho_t\beta\gamma}{\sigma_y(\gamma + 1/3)} \left[\frac{RT}{(M - b\rho_p)} - \frac{a\rho_p}{M(M + b\rho_p)\sqrt{T}} \right] \quad (22)$$

Except for the constant factor $4\gamma/(3(\gamma + \frac{1}{3}))$, this expression is the same as that derived for the spherical tank. Thus the optimum storage pressure in a cylindrical tank will be the same as in a spherical tank.

In some cases, propellant storage could be volume limited, as well as weight limited. For this reason, it is useful to examine the penalties associated with a deviation from this optimum storage density. Figure 2 shows the tankage fraction for xenon at 300 K plotted as function of xenon density. As an example, consider a reduction in the diameter of a spherical tank by 10%. This would require a 27% reduction in volume, and a 37% increase in storage density. To accomplish that requires an 87% pressure increase and a 38% increase in

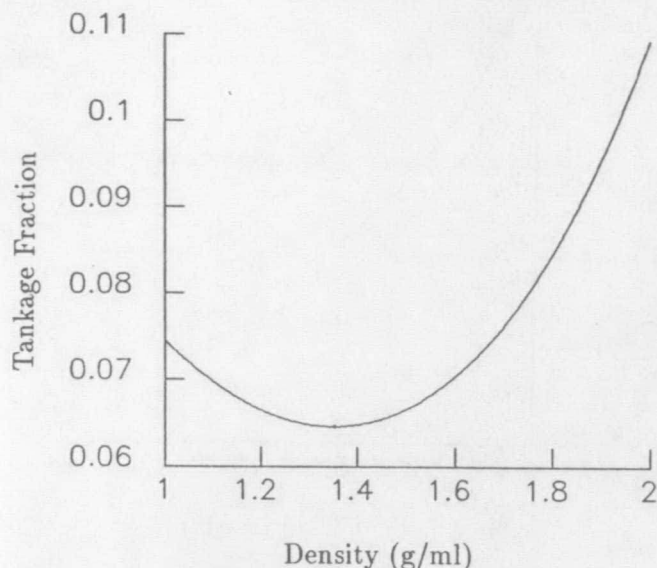


Figure 2. Tankage fraction as a function of propellant density for xenon at 300 K in a titanium tank.

tank mass. Storage at less than optimum density would result in similar tank mass increases, with no benefit. Clearly then it is important that the noble gases be stored at the proper pressure. It should be noted that, while the Redlich-Kwong equation is a good approximation to the P-V-T behavior of gases in the vicinity of the critical point, it is not exact. One should, therefore, use experimental data on the P-V-T behavior in order to determine the exact optimum storage pressure. This does not, however, change the conclusion that there is an optimum storage pressure, and that deviation from this pressure results in a significant mass penalty.

I mentioned above that the optimum storage density and the tankage fraction are both independent of total propellant mass. This means that the pressure containment mass of a single tank containing 1000 kg of propellant is equal to the total pressure containment mass of two smaller tanks, each with 500 kg of propellant. This implies that the propellant in an EP mission with supercritical storage could be distributed among a series of small spherical tanks, with only the additional plumbing and tank supports as mass penalties. This has several possible advantages. Distributed tankage may help alleviate spacecraft size constraints which might otherwise dictate higher pressure storage, requiring more massive tanks. Distributed tankage may also make it possible to use the propellant from several tanks in series, jettisoning them when empty.

In addition to pressure containment, the tank must support its own weight and the weight of the propellant

against launch loads. Typical design acceleration limits are about 6 g. The relative importance of acceleration loading can be estimated by comparing it to the pressure loading analyzed above. The pressure exerted on the bottom surface of a tank by its fluid contents due to acceleration loading is

$$P_a = \rho_p a h \quad (23)$$

where a is the acceleration and h is the height of the fluid column in the tank. If this is very small compared to the internal tank pressure, then it will not contribute significantly to the tank mass. Let us consider a couple of cases to estimate the significance of this term for high pressure storage of the noble gases. For xenon at 300 K, the optimum storage pressure is 8.3 MPa and the density is 1.35 g/ml. A 3000 kg propellant load will have a volume of 2.22 m³ and will fit into a sphere with a diameter of 1.62 m. Evaluating equation 23 with these inputs, and using a 6 g acceleration gives $P_a = 1.28$ MPa, or 1.5% of the pressure load. For an equivalent load of krypton at 300 K, the acceleration loading is only 0.5% of the pressure loading. In these cases, the acceleration loading is clearly of very small importance, but, as we will see below, the same result does not obtain in the case of cryogenic storage of these propellants. In addition to these steady launch loads, there may also be high frequency transient loads which could contribute to the failure of a tank. A quite detailed structural analysis is, of course, required for any given tank design to see if it can be expected to survive the launch environment.

When examining tank mass requirements for EP, one must also give some thought to the tank materials. Wherever the tank material properties show up in the above equations for tank mass or tankage fraction (eg equations 3, 7, 17, 22), it is always in the form of ρ_t/σ_y , the strength-to-weight ratio. In the examples above I have used titanium, but there exist other materials with a higher strength-to-weight ratio than titanium. Specifically, filament wound composite materials are currently being used extensively in high pressure tanks. The basic equations derived above are still valid, of course. One need simply use the appropriate material properties to obtain the tank mass.

Cryogenic Storage

Let us now consider the situation with cryogenic storage. The storage tank in this case must contain the propellant at a pressure which will probably be near one atmosphere, but more importantly, at a temperature well below ambient spacecraft temperatures. In examining this area, I will first develop the basic

relations for thermal balance in EP related cryogenic storage systems, I will then look at insulation mass requirements, and finally, I will consider structural mass requirements.

In any cryogenic system, there will be some heat leakage from the surroundings into the system. This heat must be removed, either by using an externally powered refrigeration system, or by letting some of the cryogen boil off, carrying the heat away as heat of vaporization. In space applications, one usually wishes to avoid the complexity and excess mass which may be associated with a refrigeration system. With electric propulsion systems based on cryogenic propellants, one would ideally design the system so that any cryogen which boils off to cool the tank is used as a propellant. This means that one would like to have a heat loading on the tank such that the boil-off rate matches the engine propellant feed requirements. It is important to note that this technique works only for EP systems in which the thruster is run continuously or quasi-continuously until the propellant is exhausted. Cryogenic storage cannot be used, for example, for a station keeping mission, without some other kind of heat sink. For solar power orbital transfer missions, where the vehicle spends a significant time in the earth's shadow with the engine not running, one must consider the propellant consumption rate, and the engine power, in terms of their average values over the entire orbit.

The most obvious result of heat loading and cryogen boil-off is that there is a limit to the lifetime of the propellant in the tank and therefore to the time period over which thrust can be available. If all the heat leaking into the tank must be carried away as heat of vaporization of the propellant, then the mass flow required to maintain the tank temperature will be given by

$$\dot{m}_T = \frac{\dot{Q}}{q_{vap}} \quad (24)$$

where q_{vap} is the heat of vaporization of the propellant, and \dot{Q} is the heat load on the tank. The value of the heat of vaporization depends on pressure, but at pressures well below the critical pressure, the dependence is small. The heats of vaporization of the various propellants at one atmosphere pressure are given in Table One. Assuming a constant heat load, the lifetime τ of the propellant is simply

$$\tau = \frac{m_p}{\dot{m}_T} = \frac{m_p q_{vap}}{\dot{Q}} \quad (25)$$

where m_p is the initial propellant mass. The lifetime of 1000 kg of propellant is shown as a function of heat loading for various propellants in Figure 3.

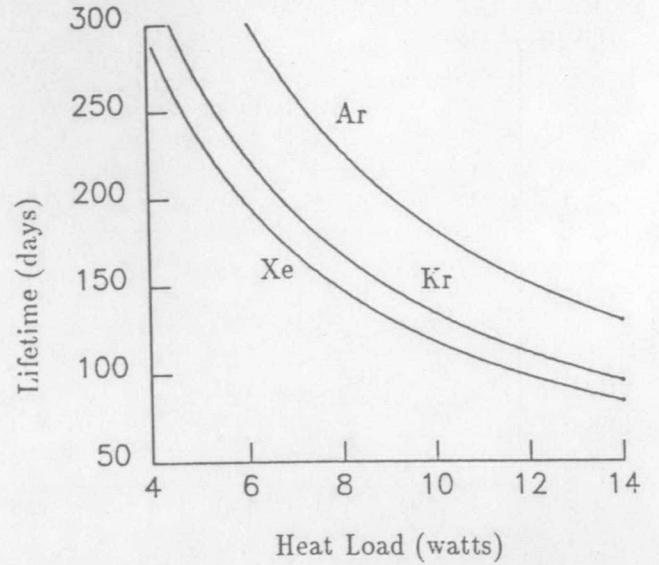


Figure 3. Orbital storage lifetime of a 1000 kg initial load of noble gas propellant as a function of heat load. The lifetime is determined by the cooling capacity of the cryogen boiling at one atmosphere pressure.

Since many EP missions under discussion today are power limited, a useful alternative method for examining the cooling requirements for cryogenic storage is to consider the power requirements of an engine capable of using all of the propellant evaporated in cooling the tank.

First, let us consider the case of the ion engine. The thrust of an ion engine is given by

$$F = \alpha \dot{m}_i v_i \quad (26)$$

where \dot{m}_i is the mass flow rate of ejected ions, v_i is the speed of the ions, and α is a beam spread factor which accounts for the loss of thrust due to components of ion velocity which are not parallel to the thrust axis (typically $\alpha \geq .97$). The power required to run an ion engine is given by

$$P = (E_i + E_n + E_k) \frac{\dot{m}_i}{m_i} \quad (27)$$

where E_i and E_n are the energy per ion to ionize and neutralize the beam, E_k is the kinetic energy per ion, and m_i is the ion mass. The specific impulse is proportional to the ratio of the thrust and the mass flow rate:

$$I_{sp} = \frac{F}{\dot{m}_T g} = \frac{\alpha \dot{m}_i v_i}{\dot{m}_T g} = \alpha \eta_p \frac{v_i}{g} \quad (28)$$

where g is the acceleration of gravity and the subscript T is used to indicate the total mass flow through the engine as opposed to the flow of ions. This distinction is necessary because some of the propellant leaks out of the engine without being ionized and therefore does not contribute to the thrust. The fraction of the atoms which are ionized is labeled η_p and is known as the propellant utilization efficiency.

The kinetic energy of the ions can be defined in terms of the specific impulse:

$$E_k = \frac{m_i v_i^2}{2} = \frac{m_i}{2} \left(\frac{g I_{sp}}{\alpha \eta_p} \right)^2 \quad (29)$$

The thrust-to-power ratio can now be defined in terms of the specific impulse:

$$\begin{aligned} \frac{F}{P} &= \frac{\alpha v_i m_i}{E_i + E_n + E_k} \\ &= \frac{m_i g I_{sp}}{(E_i + E_n) \eta_p + \frac{m_i}{2} \left(\frac{g I_{sp}}{\alpha \eta_p} \right)^2 \eta_p} \end{aligned} \quad (30)$$

In an ideal ion engine, there would be no ionization or neutralization energy loss, no beam spread, and a 100% propellant utilization efficiency. Under these conditions, this expression would reduce to

$$\frac{F}{P} = \frac{2}{g I_{sp}} \quad (31)$$

The energy required for ionization and neutralization will differ for different engine designs and probably for different propellants. In addition, there is a connection between the ionization energy and the propellant utilization efficiency. Engines have been designed with ionization costs of 90 W/A and propellant utilization efficiencies greater than 0.9.⁸ More recent engines designed for long life have ionization costs around 120-130 W/A for propellant utilization efficiencies around 0.9.⁹ It is expected that ionization cost will go up and utilization efficiencies will go down with the lighter gases (krypton and argon), but this effect should be smaller than 10%. Expected thrust-to-power ratios for the noble gases are shown graphed as a function of specific impulse in Figure 4 for selected values of E_i and η_p .

The power in the beam of an ion engine is given by

$$P_B = \frac{1}{2} \dot{m}_i v_i^2 \quad (32)$$

and the power required to run the engine is

$$P_e = \frac{P_B}{\eta_e} \quad (33)$$

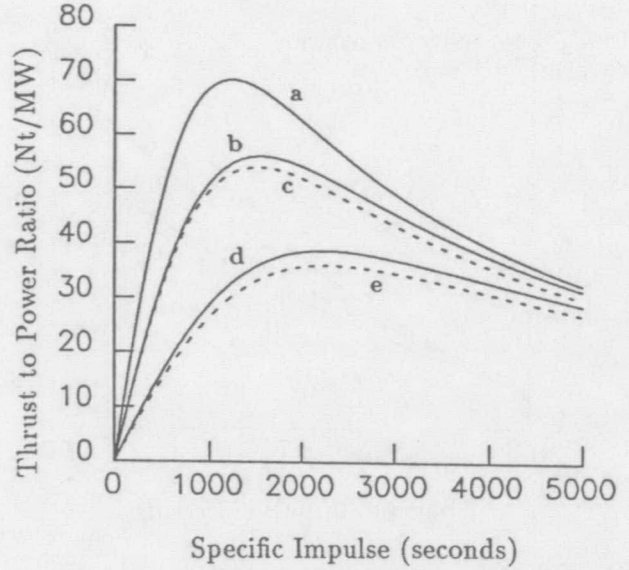


Figure 4. Thrust-to-power ratio for ion engines.

- a: Xe, $E_i = 120$ W/A, $\eta_p = .90$
- b: Kr, $E_i = 120$ W/A, $\eta_p = .90$
- c: Kr, $E_i = 130$ W/A, $\eta_p = .85$
- d: Ar, $E_i = 120$ W/A, $\eta_p = .90$
- e: Ar, $E_i = 140$ W/A, $\eta_p = .85$

where $\eta_e = E_k / (E_k + E_i + E_n)$ is defined as the electrical efficiency of the engine. The ratio between the engine power and the heat loading on the tank is then

$$\frac{P_e}{\dot{Q}} = \frac{g^2 I_{sp}^2}{2 q_{vap} \alpha^2 \eta_p \eta_e} \quad (34)$$

where the exhaust velocity has been expressed in terms of the specific impulse. This expression is still not in its most useful form; one usually wishes to consider variations with specific impulse. This dependence is explicit only when the electrical efficiency is rewritten to include its dependence on I_{sp} :

$$\begin{aligned} \eta_e &= \frac{E_i + E_n + E_k}{E_k} \\ &= \frac{E_i + E_n}{E_k} + 1 \\ &= \frac{2(E_i + E_n)}{m_i} \left(\frac{\alpha \eta_p}{g I_{sp}} \right)^2 + 1, \end{aligned} \quad (35)$$

so

$$\frac{P_e}{\dot{Q}} = \frac{(E_i + E_n) \eta_p}{q_{vap} m_i} + \frac{g^2 I_{sp}^2}{2 q_{vap} \alpha^2 \eta_p} \quad (36)$$

In an ideal engine, this reduces to

$$\frac{P_e}{\dot{Q}} = \frac{g^2 I_{sp}^2}{2 q_{vap}} \quad (37)$$

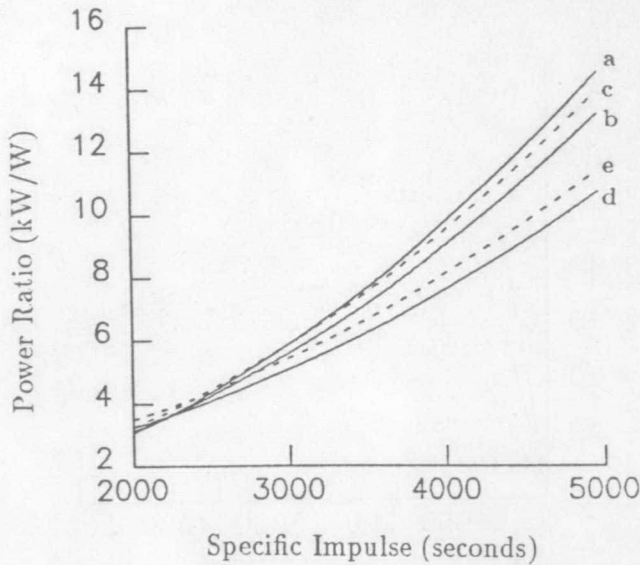


Figure 5. The engine power in kW required to use all of the cryogen boiling off to cool a tank under a one watt heat load.

- a: Xe, $E_i = 120 \text{ W/A}$, $\eta_p = .90$
- b: Kr, $E_i = 120 \text{ W/A}$, $\eta_p = .90$
- c: Kr, $E_i = 130 \text{ W/A}$, $\eta_p = .85$
- d: Ar, $E_i = 120 \text{ W/A}$, $\eta_p = .90$
- e: Ar, $E_i = 140 \text{ W/A}$, $\eta_p = .85$

Using representative values for the efficiencies of non-ideal engines, the power ratios have been calculated as a function of specific impulse for several different cases and are plotted in Figure 5. At 2000 seconds, the minimum engine power will need to be 3000 to 3500 times the tank heat load to avoid wasting propellant; for a 30 kW ion engine, tank heat loads up to 8.5 to 10 watts will be acceptable. At 5000 seconds, the power ratio is much more restrictive, requiring 10 to 15 kW of engine power per watt of tank heat load. For a 5000 second 30 kW ion engine, tank heat loads will need to be limited to 2-3 watts. These results will, of course, correspond to the propellant flow rates and maximum trip times discussed above.

For arcjets, the final expression will be simpler, if only because it is more difficult to break down the inefficiencies of arcjets into discreet and easily identifiable components. Thus we have that the specific impulse of an arcjet is related to the thrust by

$$I_{sp} = \frac{F}{\dot{m}g} \quad (38)$$

The efficiency of an arcjet is defined as the ratio of the thrust power to the total electric power into the engine:

$$\eta = \frac{F^2}{2P_e \dot{m}} = \frac{I_{sp}^2 \dot{m} g^2}{2P_e} \quad (39)$$

Recalling that $\dot{Q} = \dot{m}q_{vap}$, we can immediately write the power-to-heat load ratio for an arcjet:

$$\frac{P_e}{\dot{Q}} = \frac{I_{sp}^2 g^2}{2\eta q_{vap}} \quad (40)$$

For an arcjet operating on hydrogen at 1200 seconds specific impulse and an efficiency of .35, the power ratio is 440 watts to the engine for each watt of heat leakage into the tank. On the other hand, because of the much lower heat of vaporization, a helium arcjet operating at 1200 seconds and .7 efficiency, will need a power ratio of 4700.

Heat leakage into the tank can come from two sources, either by radiative heat transfer from the tank surroundings, or by thermal conduction along the tank structural supports and tank penetrations. Radiative transfer is controlled by multi-layer insulation (MLI) blankets and in severe cases with vapor cooled shields. Thermal conduction is limited by making the supports and penetrations of low thermal conductivity materials, and by using vapor cooled heat intercepts along the conduction paths. A detailed tank design will be required in order to allocate the total heat leakage between these two paths, but for a first estimate, it is useful to assume that the radiative heat load must be absorbed by heat of vaporization of the propellant, and that the heat load along conduction paths is absorbed by the heat capacity of the vapor as it is warmed to ambient temperature. This assumption is a little bit generous in that it requires that the heat intercept along the conduction paths be perfect so that there will be essentially zero net heat input to the tank from the conduction load. This is, of course, not possible, but it can be approached if necessary, and the remaining conduction heat load can be balanced by reducing the allowed radiative heat load. Additionally, in severe cases, as mentioned, the thermal protection system can include vapor cooled shield(s) within the MLI blanket. This results in some of the radiative heat load being absorbed by the heat capacity of the warming gas, rather than through the heat of vaporization. For a first analysis, however, we will assume that the radiative heat load on the tank should match the heat load required to vaporize the propellant at the proper flow rate.

An estimate of the insulation mass required to achieve this result can be obtained by considering as a figure of merit the product of density ρ_i and thermal conductivity κ of the insulation. Walburn¹⁰ describes a goldized Kapton multi-layer insulation system which has been ground tested and shown to have $\rho_i \kappa = 6.44 \times 10^{-5} \text{ (Btu lb)/(hr ft}^4 \text{ R)}$, or in mks units $1.79 \times 10^{-3} \text{ (W kg)/(m}^4 \text{ K)}$. Thermal conductivity is

defined as

$$\kappa = \frac{\dot{q}d}{A\Delta T} \quad (41)$$

where d is the thickness of the insulation, A is the surface area, \dot{q} is the heat flux, and ΔT is the temperature difference. Thus

$$\rho_i \kappa = \frac{\rho_i \dot{q}d}{A\Delta T} = \frac{m_i \dot{q}}{A^2 \Delta T} \quad (42)$$

where m_i is the mass of the insulation. Rearranging gives

$$m_i = \frac{\rho_i \kappa \Delta T A^2}{\dot{q}} \quad (43)$$

In this expression, we have the square of the surface area, rather than the volume of the tank. This means that, unlike the structural mass, the insulation mass cannot be written as a fraction of the initial propellant load. Because the surface area to volume ratio decreases as the total volume increases, the insulation mass fraction will be smaller for larger tanks.

Let us consider a few cases. We saw above that for an ion engine operating on xenon at 5000 seconds I_{sp} at 30 kW, the tank heat load will need to be limited to 3 W. Liquid xenon has a density of 3.08 kg/l at 165 K, so if we assume an initial 3000 kg propellant load, the tank will need a volume of 974 l. A spherical tank of this volume will have a radius of .61 m and a surface area of 4.75 m². Assuming an ambient temperature of 300 K, the temperature difference will be 135 K. Equation 43 then gives us an insulation mass of only 1.8 kg. There are some obvious difficulties with this result, the main one being that the density of the MLI blanket is probably not independent of thickness. In this case, because of the small area and the small ΔT , the insulation requirements are not very strong, and the blanket will be very thin. It will probably take more than 1.8 kg just to make the blanket structurally sound and attach it over the entire area of the tank. In any case, it appears that for this particular mission, the insulation requirements for cryogenic xenon storage contribute very little to the tankage fraction.

For a 15 kW hydrogen arcjet operating at 1200 seconds and an efficiency of .35, or for a 30 kW hydrogen arcjet operating at 1200 seconds and an efficiency of .35 with a 50% duty cycle, the allowable tank heat load will be 34 W. Liquid hydrogen has a density of 67 kg/m³, at a temperature of 20 K. A 3000 kg initial propellant load will have a volume of 44.8 m³, which will fit into a sphere with radius 2.2 m and surface area of 61 m². Using $\Delta T = 280$ K in equation 43, we get an insulation mass of 55 kg, or about 1.8% of the initial propellant load.

The same arcjet using helium at 1200 seconds and an efficiency of .7 would require a heat load of only 3.2 W. Liquid helium, at 124.8 kg/m³, is denser than hydrogen, and would require a tank with a volume of only 24 m³. A spherical tank with this volume would have a radius of 1.8 m and a surface area of 40 m². The temperature difference is slightly larger in this case; $\Delta T = 296$ K. We get an insulation mass for this tank of 265 kg, significantly higher than for hydrogen, but still only 8.8% of the initial propellant mass.

Finally, we can consider the structural mass of these tanks. In the above analyses, I have assumed that the propellants were stored at their normal boiling points, requiring pressure containment at one atmosphere. For a spherical tank, the tank mass is given by equation 3:

$$m_t = 4\pi r^2 t \rho_t = \frac{3P\beta m_p \rho_t}{2\sigma_y \rho_p} \quad (3)$$

For a fixed propellant mass and storage pressure, the tank pressure containment mass is inversely proportional to the propellant density. For the cryogenic case, one needs a somewhat larger safety factor because of possible temperature excursions, so using $\beta = 3$, and $P = 1 \text{ atm} = 101 \text{ kPa}$, we find that for a 3000 kg initial propellant load in a titanium tank, xenon will require only 1.5 kg for pressure containment, helium will require 38 kg, and hydrogen will require 70 kg for pressure containment. Of course, we must also consider acceleration loading. In equation 23, we show that one can treat acceleration loading in terms of an equivalent pressure. For the three cases under discussion here, we have acceleration pressures (at 6 g) of 230 kPa for xenon, 26.4 kPa for helium, and 17.3 kPa for hydrogen. We see here that acceleration loading will dominate for xenon, while pressure loading will dominate for helium and hydrogen. If we assume that the two forms of stress add linearly, the structural masses become 5.0 kg for xenon, 48 kg for helium, and 83 kg for hydrogen.

An additional point to keep in mind when designing the propellant tank is the feed pressure requirements of the engine. If the storage pressure is lower than the engine feed pressure, then there will be a need for a compressor in the feed line. Ion engines typically operate at low flow rates and feed pressures well under one atmosphere, while arcjets are inherently high pressure devices, requiring inlet pressures between 2 and 10 atmospheres. In the above analysis, I assumed that cryogenic propellants would be stored at one atmosphere pressure. Clearly, for the noble gases this is no problem, but for hydrogen and helium, an in-line compressor would have to be used to allow storage at this

pressure. On the other hand, we can examine the implications of higher pressure cryogenic storage. For the purposes of analysis, let us consider cryogenic storage at 5 atmospheres pressure. The most obvious result is that the tank structural mass has to increase to accommodate the increased pressure. In the case of hydrogen, two additional things happen, both of which lead to somewhat more massive tanks. At the higher pressure, the hydrogen must be somewhat warmer in order to vaporize (which is necessary if one wishes to use heat of vaporization to cool the tank). This higher temperature results in a somewhat lower density and a larger tank. Additionally, at the higher temperature and pressure, the heat of vaporization is decreased, reducing the cooling capacity of the hydrogen and increasing the requirements for insulation mass. Given the exact values of temperature, density, and heat of vaporization of hydrogen at 5 atmospheres pressure, one can follow the procedures described above to determine the resulting tank mass. One can then evaluate the mass and reliability tradeoffs involved in choosing between high pressure storage and using a compressor in the feed line.

For helium, the situation is quite different. Five atmospheres pressure is above the critical pressure of helium, and one has, in effect, both cryogenic and supercritical storage at the same time. There is no longer any phase change possible, and the heat of vaporization goes to zero. It is no longer possible to cool the tank by vaporizing the propellant. This is not to say that it is impossible to store helium under these conditions: the Apollo Lunar lander used supercritical cryogenic helium to pressurize the descent engines.³ In this case, one must match the heat load with an allowable expansion rate, which would correspond to the arcjet propellant consumption rate. A detailed analysis of the thermodynamic properties of helium in this pressure and temperature range would be necessary to determine tankage fraction for this case.

Conclusion

We have shown that when the problems specific to propellant storage for electric propulsion are examined in detail, there appear to be no fundamental roadblocks to obtaining reasonable tankage fractions for any of the propellants of interest for EP missions, including helium. In order to achieve these tankage fractions, one must pay attention to certain principles applicable to EP missions. In cryogenic storage applications, one must use the propellant to cool the tank before it is fed to the engine, and in supercritical storage, one must be aware that there are optimum storage pressures that differ from one gas to another, and that even small de-

viations from these pressures can result in significant tankage fraction penalties.

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References

1. G.E. Roos, et al. "Electrothermal Engine Propellant Storage and Feed System Study," NASA-CR-52, May, 1964.
2. A.A. Fowle, et al. "Cryogenic Propellant Feed Systems for Electrothermal Engines," NASA-CR-60, May, 1964.
3. M.L. Davis, R.K. Allgeier, T.G. Rogers, and G. Rysavy, "The Development of Cryogenic Storage Systems for Space Flight," NASA-SP-247, 1970.
4. P.J. Lawlor and K.M. Montgomery, "Research and Development of Propellant Feed Systems for Ion Engines," NASA-CR-51, May, 1964.
5. P.J. Lawlor, "Lithium Feed Systems for Arc Jet Engines," TRW ER-5444, March 2, 1964.
6. R.P. Welle, "Availability Considerations in the Selection of Inert Propellants for Ion Engines," AIAA-90-2589, AIAA/DGLR/JSASS 21st International Electric Propulsion Conference, Orlando Florida, 18-20 July, 1990.
7. *Materials in Design Engineering*, October, 1963 pp 162-3.
8. J. S. Sovey, "Improved Ion Containment Using a Ring-Cusp Ion Thruster," *J. Spacecraft*, Vol. 21, No. 5, Sept.-Oct., 1984, p 488-495.
9. J. R. Beattie and J. N. Matossian, "Xenon Ion Propulsion for Stationkeeping and Orbit Raising," IPEC-88-052, DGLR/AIAA/JSASS 20th International Electric Propulsion Conference, Garmisch-Partenkirchen, Germany, 3-6 October, 1988.
10. Allen B. Walburn, "Reusable Multilayer Insulation Development," *J. Spacecraft*, Vol. 12, No. 11, November 1975 p 701-703.